



HADRON ELASTIC SCATTERING - AN EXPERIMENTAL REVIEW^{*}

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April 1977

^{*}Invited talk presented at the XIIth Rencontre De Moriond, March, 1977



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ABSTRACT

Recent data on the elastic scattering of hadrons is summarized. Particular emphasis is given to the region of small four-momentum transfer.

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The last few years has seen a significant increase in the amount of high energy hadron elastic scattering data available to us. Not only have we new data in regions of s and t not previously accessible but more familiar regions of these variables have been revisited by much more precise experiments. I would like to review from an experimentalist's point of view this new data and how it modifies or extends our picture of high energy elastic scattering.

It is for pp elastic scattering that we have probed the furthest in both t and s . This is not because pp scattering is more interesting than πp , but because we have had available much more intense beams of protons than other particles. Because of this and the truly unique nature of the ISR we know a great deal more about pp elastic scattering than any other reaction. Figure 1 is my attempt to depict schematically the essential features of the pp differential cross section at a laboratory energy of about 2 TeV, equivalent to the highest ISR energy. Existing data span a t range from the Coulomb region ($t \approx -0.001 \text{ GeV}^2$) to $t \approx -8 \text{ GeV}^2$ and about a dozen order of magnitudes in cross section. Before we begin to feel too pleased with ourselves however we should recall that a 90° scatter at 2 TeV represents a four momentum transfer of 1875 GeV^2 so we have really explored a very limited part of the kinematical range available to us at high energies.

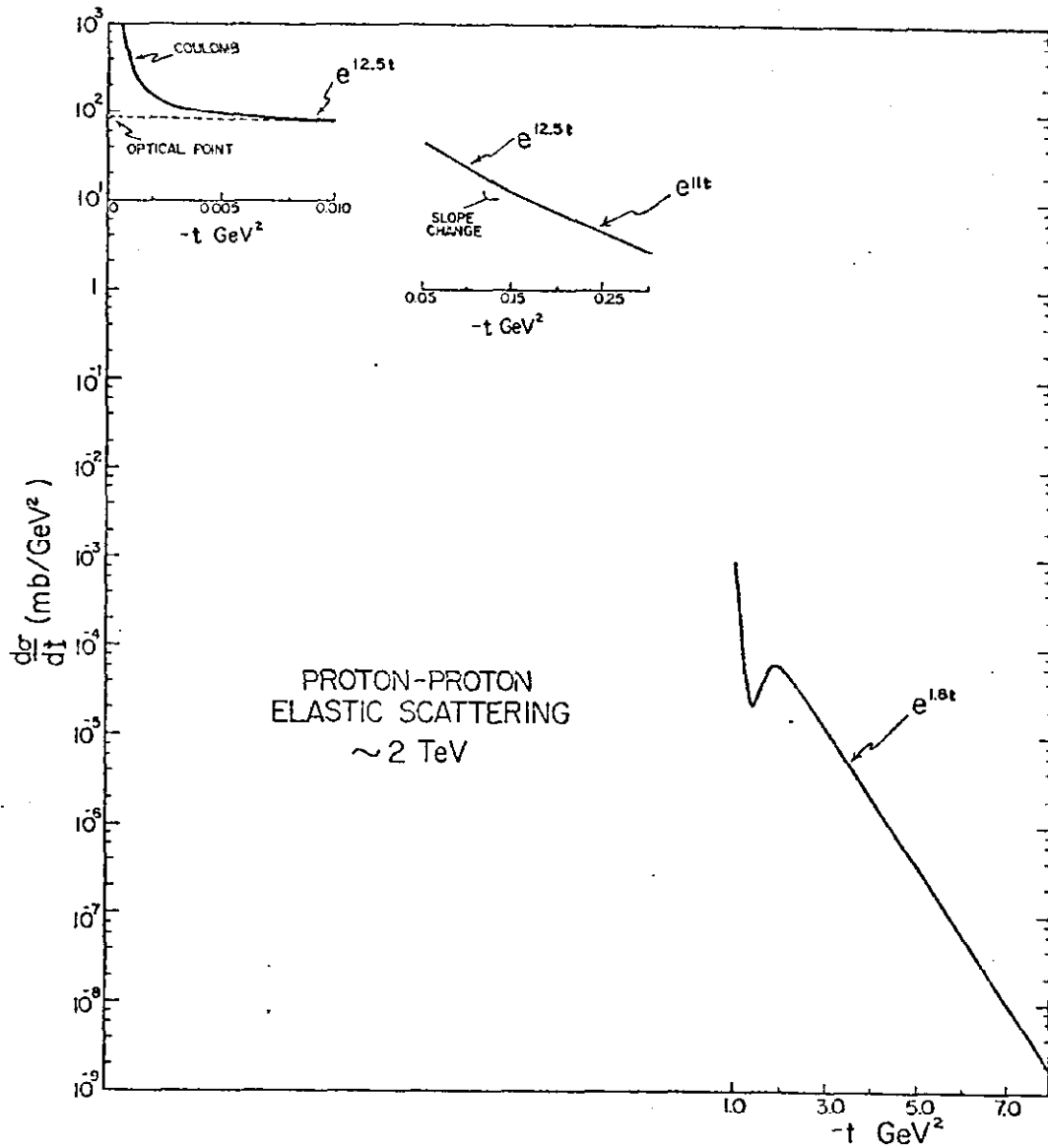


Figure 1: General feature of pp elastic scattering at 2 TeV.

By way of orientation let us look at the main features of Fig. 1. In the very smallest t region we see the steeply rising pure Coulomb peak ($\sim 1/t^2$) and the forward nuclear portion which is well described by a linear decrease on a logarithmic scale. The region at $t \approx -0.002$ is an important one because the nuclear and Coulomb contributions are comparable and their interference can be used to measure $\rho = \text{Re } f(\alpha) / \text{Im } f(\alpha)$

Continuing out to larger t we find at about $t = -0.14$ a change of slope by about 1.5 units. Following this slope change the next feature is a pronounced dip at $t \approx -1.4 \text{ GeV}^2$ and then a much more shallower slope which continues out to as far as measurements exist. I'd now like to examine each of these regions in the light of recent data.

In the Coulomb-nuclear interference region there are new precise measurements of ρ at the highest ISR energies. Recall that dispersion relations relate ρ to integrals over the pp and $\bar{p}p$ total cross sections as a function of energy, so precision measurements of ρ give us glimpses of total cross section behavior well beyond energies attainable with today's accelerators.

At high energies ρ is not very large and its precise measurement is a challenge to the experimentalist. Recent data from the CERN-Rome group in Fig. 2 illustrates this. The top drawing shows the magnitude of the effect that

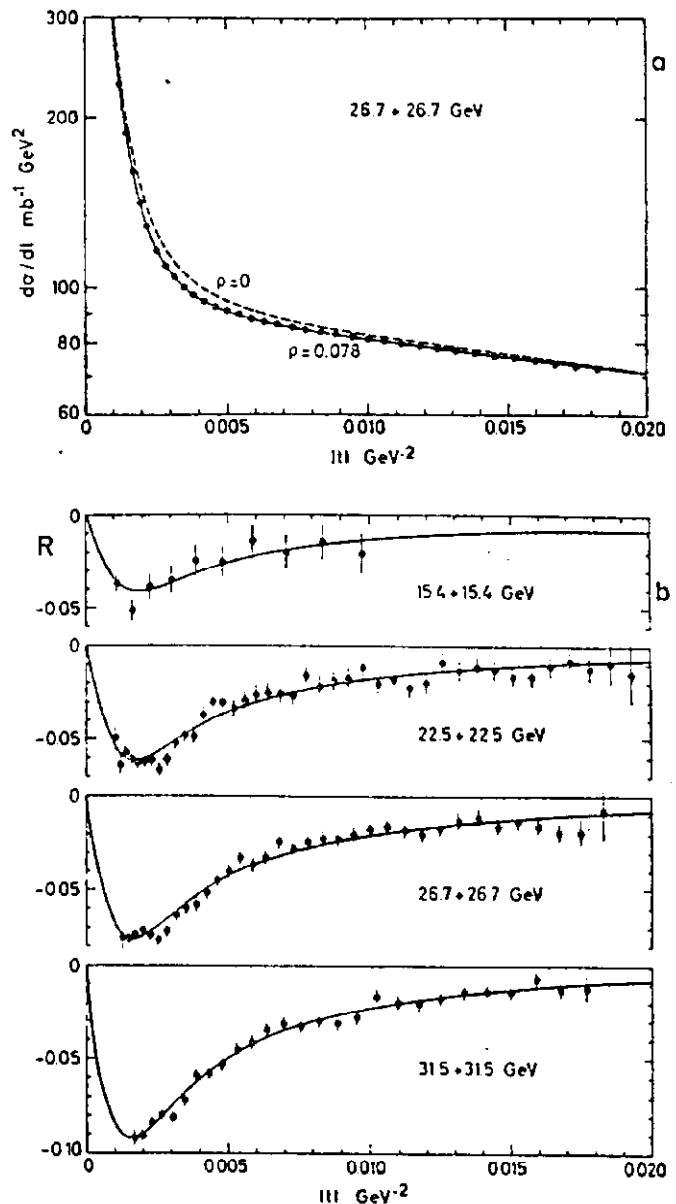


Figure 2: Recent ISR results¹ on pp scattering in the Coulomb Region. The parameter R is defined in the text.

is being measured. Plotting the quantity R defined as

$$R = \frac{d\sigma/dt \text{ (measured)}}{d\sigma/dt \text{ (for } \rho = 0)} - 1.$$

illustrates the sensitivity of this experiment. The curves in Fig. 2b show their best fits to ρ for each of their energies. This experiment has very good statistics and measured ρ to a precision of 0.01. Figure 3a shows the momentum dependence of ρ and Fig. 3b shows the total cross section data used as input to the dispersion relation predictions also shown in Fig. 3a. This group has performed a simultaneous fit to both the measurements of ρ and the

total cross sections. The dashed regions in figures

3a and 3b indicates the one standard deviation region for this simultaneous fit. It is a nice consistent presentation of the dispersion relation predictions and the CERN-Rome group conclude that the pp total cross section continues to increase until at least 40 TeV.

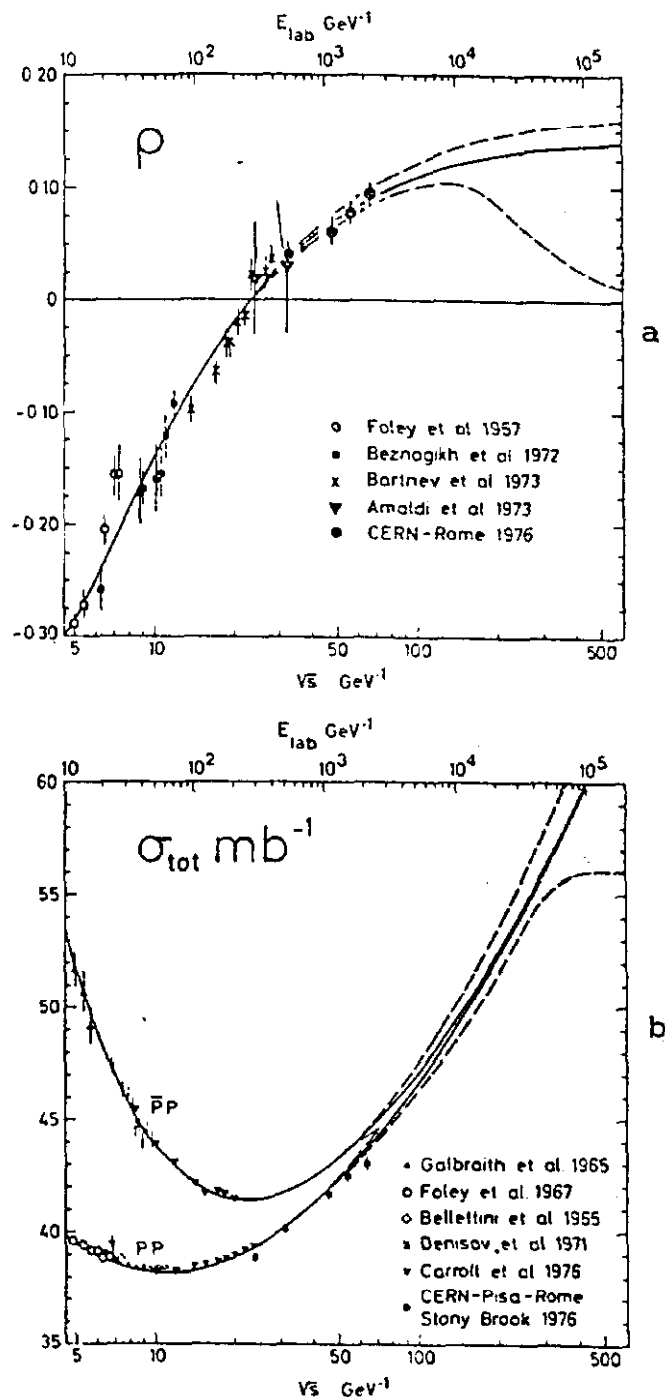


Figure 3: Simultaneous fit to ρ and σ_{tot} from Ref. 1. The dashed lines represent the one sigma limits for ρ and σ_{tot} .

Measurements of ρ for the other hadrons, π^\pm , K^\pm and \bar{p} are not nearly as extensive nor cover a comparable s range. Hendrick and Lautrup² in 1975 using the then newly available Fermilab total cross sections computed ρ as a function of s via dispersion relations and made comparisons with existing data. I would like to update their comparisons with the significant amount of new data now available. The data come from recent experiments at SLAC,³ CERN,⁴ Serpukhov⁵ and Fermilab.⁶ Figure 4 shows the dispersion relation predictions and the now available data for $\pi^\pm p$. Recent data not included in the Hendrick and Lautrup comparison is indicated by arrows. Except for a small region around 30 GeV/c in the $\pi^- p$ case the agreement seems good. There are a number of fine points in the calculation of the real parts from the total cross section data and I refer those interested to the work of the Karlsruhe group.⁷ The Fermilab points represent a preliminary analysis on a partial sample and the final results will have considerably smaller errors. As required for total cross sections which are rising, ρ does appear to become positive in the Fermilab energy region.

Figure 5 shows ρ measurements for $K^\pm p$ and is taken from a paper of Baillon, et al.,⁴ reporting their new results at 4.2, 7 and 10 GeV/c. The dispersion relation prediction also shown is theirs but agrees well with Hendrick and Lautrup.² The main difference between Fig. 5 and similar figures shown by Hendrick and Lautrup² is that there were serious discrepancies between their calculations and the data then at hand. The new high statistics counter data^{3,4} is in much better agreement with theory as is the revised 14.3 GeV/c $K^- p$ bubble chamber point of De Boer, et al.,⁸ which I have added to Fig. 5.

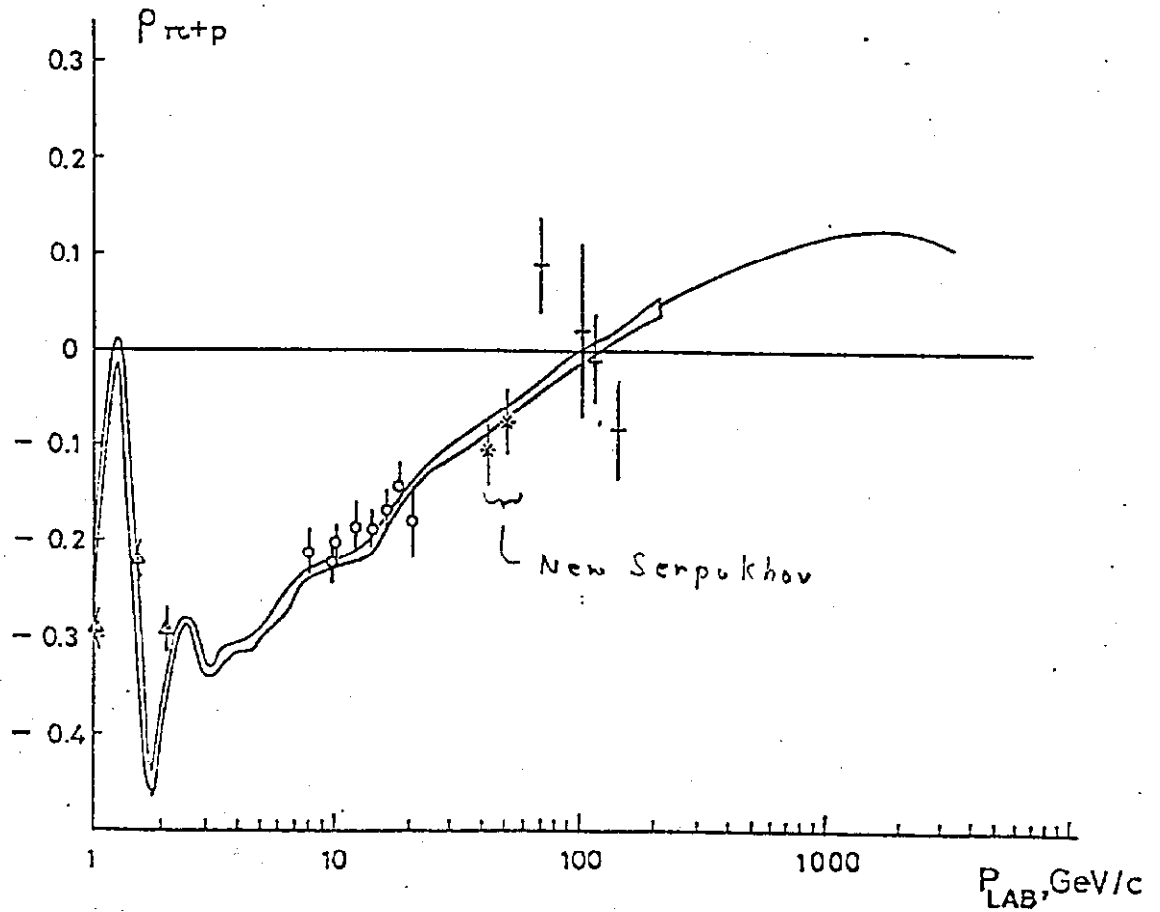
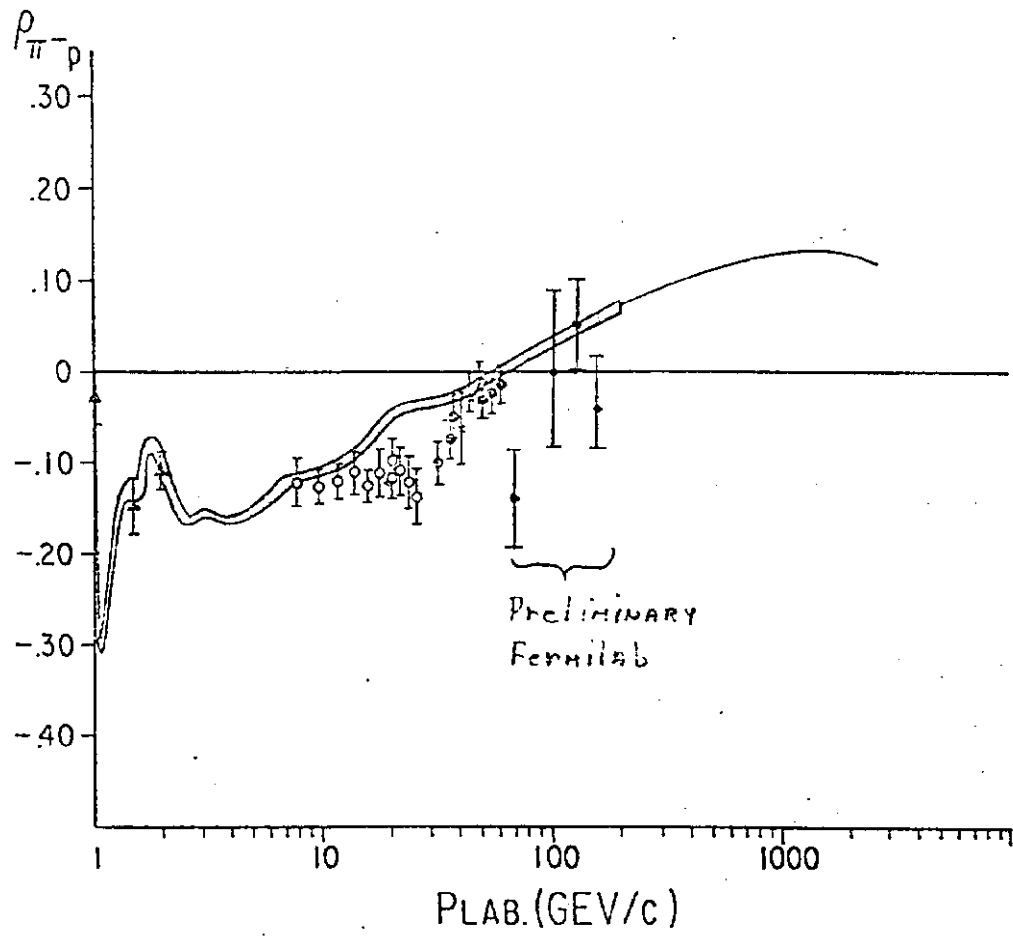


Figure 4: Summary of ρ measurements for $\pi^\pm p$.

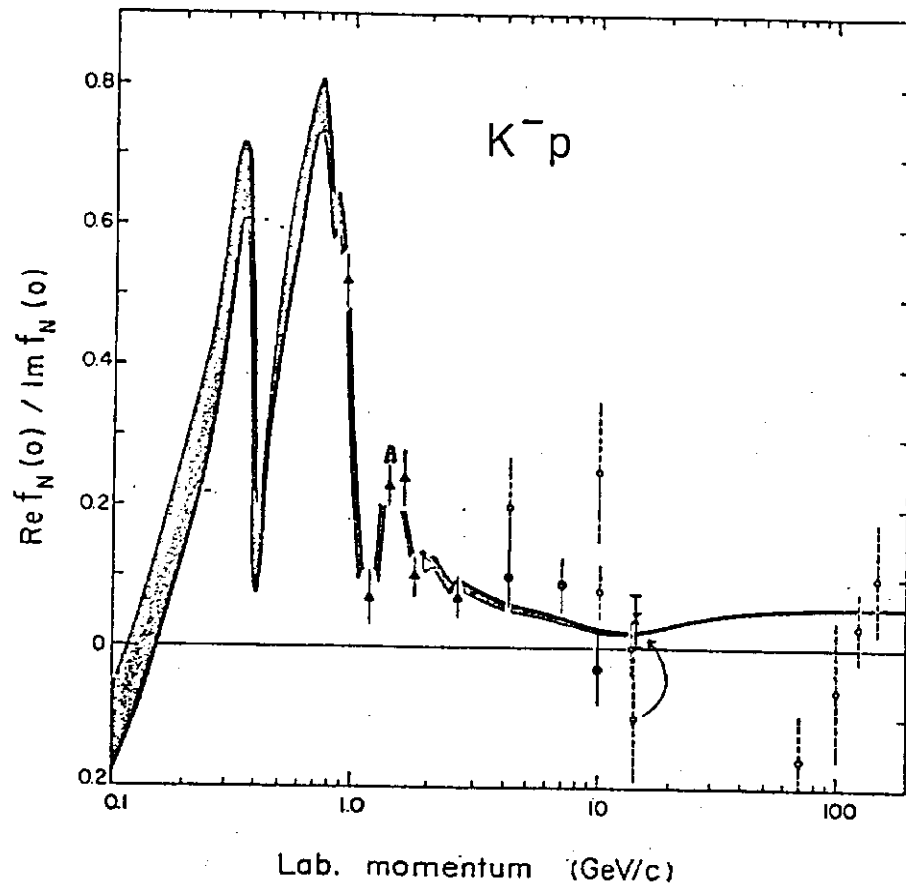
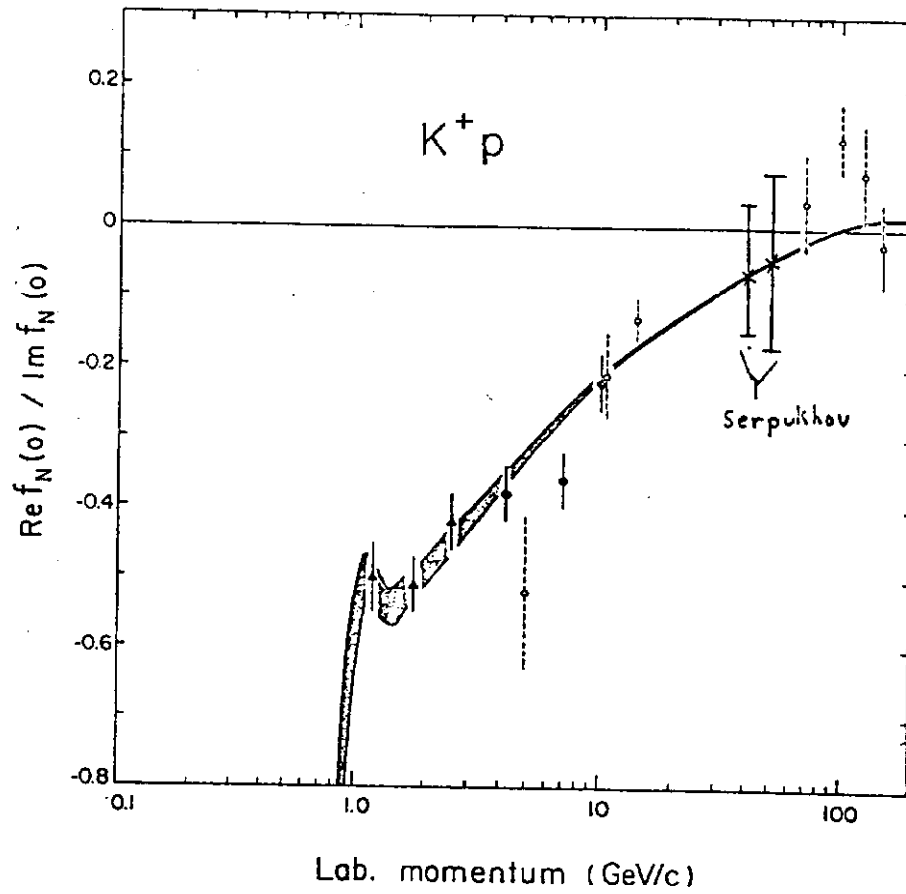


Figure 5: Summary of ρ measurements for $K^\pm p$.
The basic figures are from Ref. 4.

For completeness, Fig. 6 shows ρ for $\bar{p}p$ and is taken from Ref. 3. The dispersion relation predictions are again those of Hendrick and Lauthrop and I have added the preliminary Fermilab data points. Considerable progress has been made in the last two years as attested by the fact that the comparable figure in the paper of Hendrick and Lauthrup had only one data point.

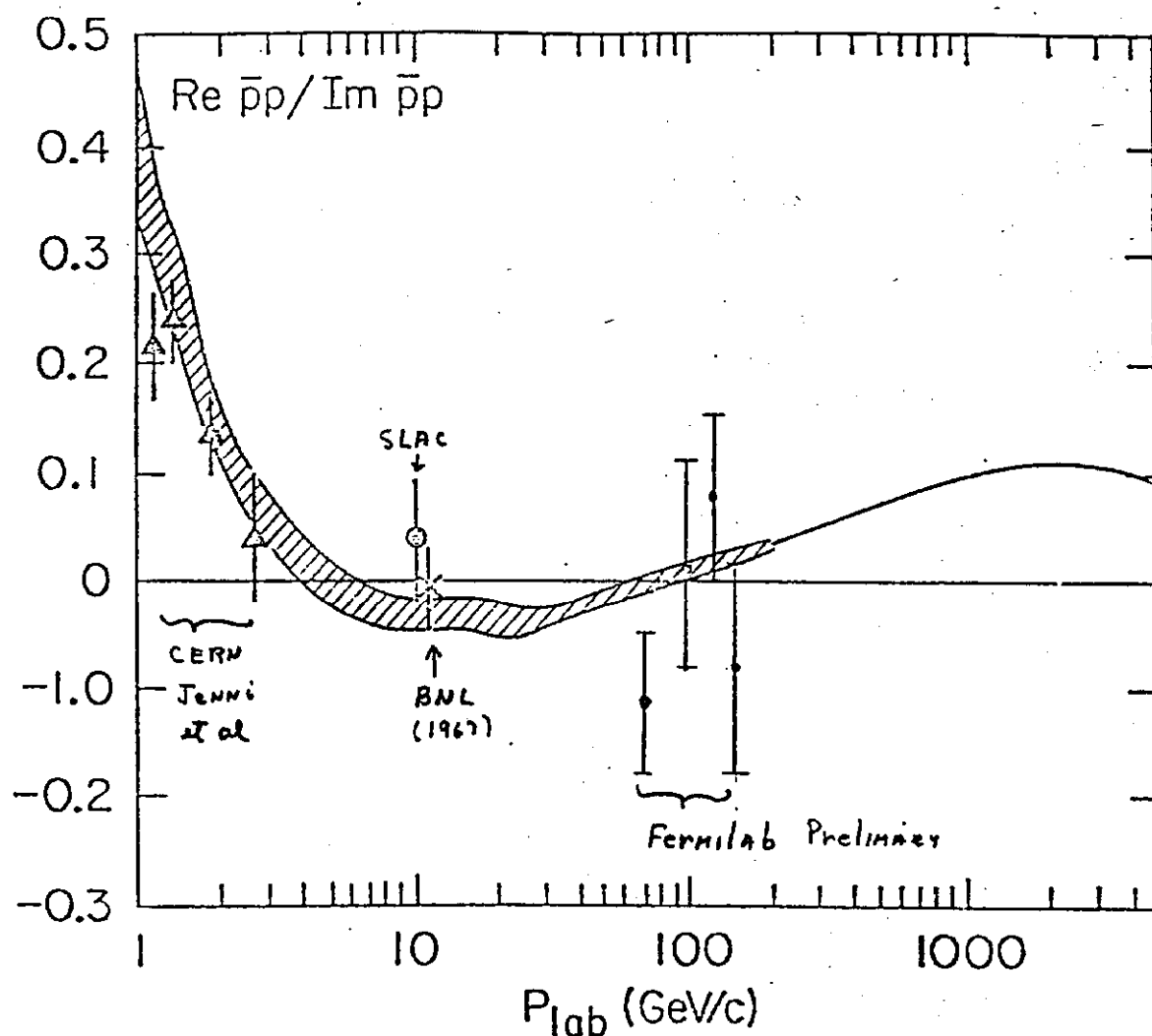


Figure 6: Summary of ρ measurements for $\bar{p}p$.

In summary the last two years have seen new data which are in much better agreement with dispersion relations. The new precision measurements of the CERN-Rome group cover the full ISR range and are of such an accuracy that further improvements, if feasible, would necessitate re-examining the higher order electromagnetic corrections which begin to be significant. New data on π^\pm , K^\pm and \bar{p} reactions are also in much better agreement with theory; we look forward to the final Fermilab results, results from the new SPS experiment and a Serpukhov experiment which has completed data taking.

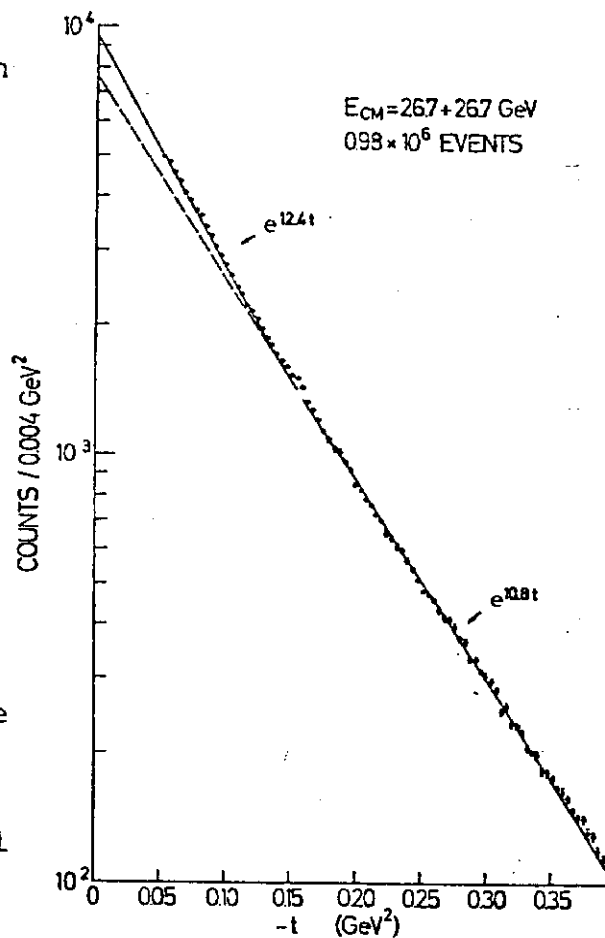


Figure 7: $d\sigma/dt$ from Barbiellini, et al.¹⁰

Interest in the region of $t \sim -0.14$ started in 1969 when Carrigan⁹ studied slope determinations by experiments in different t ranges and concluded that there must be a slope change in that region. However it was not until the high statistics experiment of Barbiellini, et al.,¹⁰ done at the ISR that this slope change showed up clearly. This is shown in Fig. 7. Experimentally it is difficult to demonstrate whether this is a break or a smooth slope change.

Preliminary results¹¹ from Fermilab E-69 are presented in Fig. 8. E-69 was primarily an experiment to measure ρ but a special run was taken at 200 GeV/c with a modified acceptance to collect more events near this slope change. A good way to

look at the data is to extract the logarithmic derivative, $b(t) = d/dt (\ln d\sigma/dt)$ as a function of t . This requires very good statistics and Fig. 9 indicates how various expressions for this slope change might look in such a plot. Looking at the pp data it is apparent that $b(t)$ changes about two units in a t interval of less than 0.05 GeV^2 . Whether one wishes to call this a break is somewhat a matter of taste but it cannot be explained by a constant curvature, $\exp(ct^2)$, factor. The behavior of the $\pi^\pm p$ data is inconclusive because of poorer statistics. It is clear however, that they have substantial curvature and that one should have at least a million events to play these games.

If one tries to compare the E-69 data for pairs of particles one finds that in this

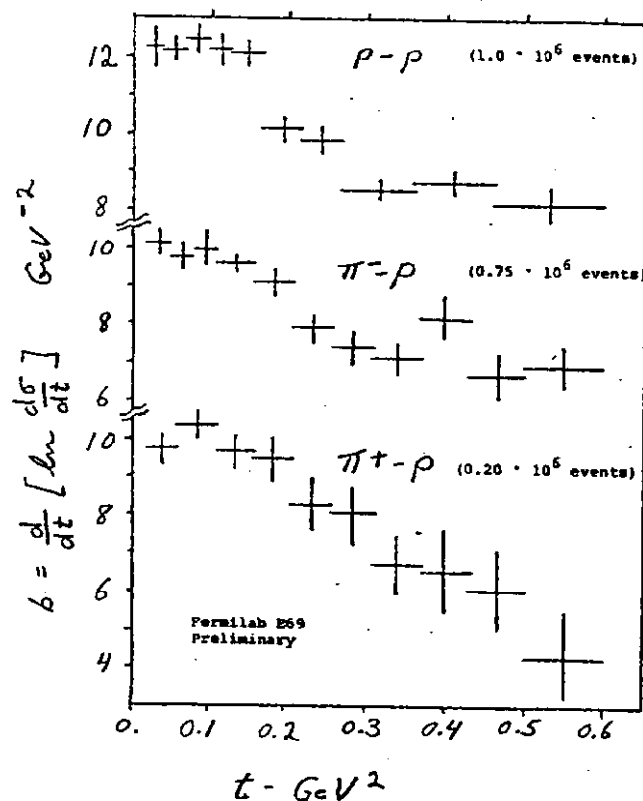


Figure 8: $b(t)$ for $\pi^\pm p$ and pp at 200 GeV/c from Ref. 11.

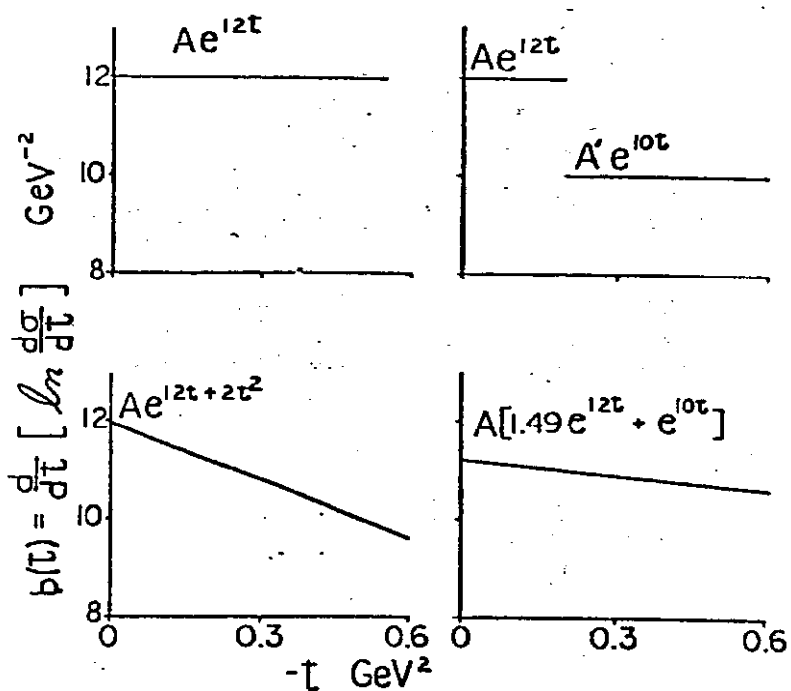


Figure 9: Various forms of the logarithmic derivative $b(t)$.

t region they differ by a simple exponential factor of t. This is illustrated in Fig. 10. Here the ratio were fitted to a simple exponential which is then divided out. It would be amusing if the complicated t structure were similar for all particles after a simple exponential term were removed.

A Columbia-Stonybrook group working at the Fermilab Internal Target area has recently looked at the s dependence of this slope change.¹² Using a polyethylene target and making a carbon subtraction they were able to fit the logarithmic slope at two t regions near the break as shown in Fig. 11. The slope difference between the two regions does not change very much even though the overall slope does change significantly over this wide s range.

A high statistics SLAC experiment³ was one of the first to measure slopes as a function of t. Their results are shown in Fig. 12 and one sees that these reactions exhibit a diverse and rather rich

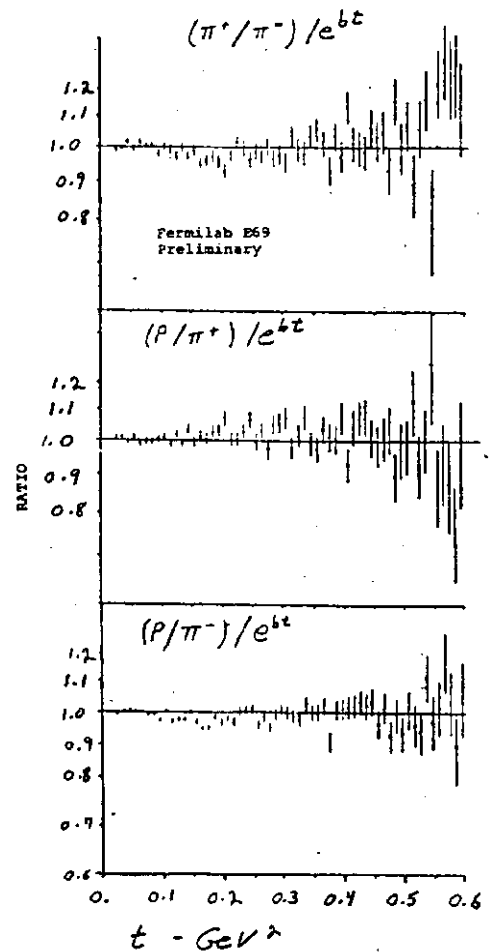


Figure 10: Ratios of $d\sigma/dt$ for π^+p and pp from Ref. 11.

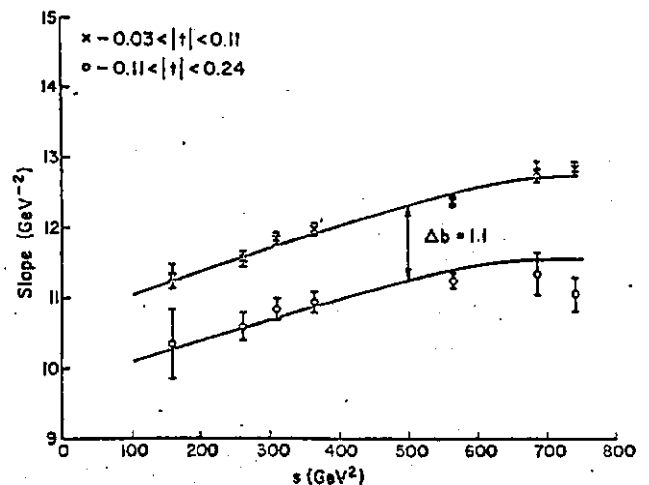


Figure 11: b determined from two different t ranges from Ref. 12.

structure not previously appreciated. Note that the $\bar{p}p$ slope actually decreases at small t . The SLAC Group has been able to fit their data to a form which contains a Regge exchange contribution and two Pomeron terms. One Pomeron component is central and the other is peripheral in impact parameter space and grows with energy. It is this term which is responsible for the small t structure. Clearly this t region is a complex one and only for the pp case do we have an appreciable amount of data.

The t region just beyond the slope change we have been discussing is where most experiments have been done in the past. The region is well covered for all the hadrons by the Fermilab single arm spectrometer group.¹³ Figures 13 and 14 illustrate their basic measurements. While looking at this data it is worth noting some obvious but important points. The data for all particles is very smooth. There are no bumps, wiggles or rapid slope variations. The data does require a t^2 term and is consistent with our earlier discussion of a slope change at $t = -0.14 \text{ GeV}^2$ but can shed no light on it because this experiment does not have sufficient statistical accuracy in the small t region. As shown in figures 13 and 14, the data is well fit by the form $d\sigma/dt = A \exp(bt + ct^2)$. Another Fermilab experiment (E7)¹⁴ covering roughly the same t region showed evidence for a more rapid curvature

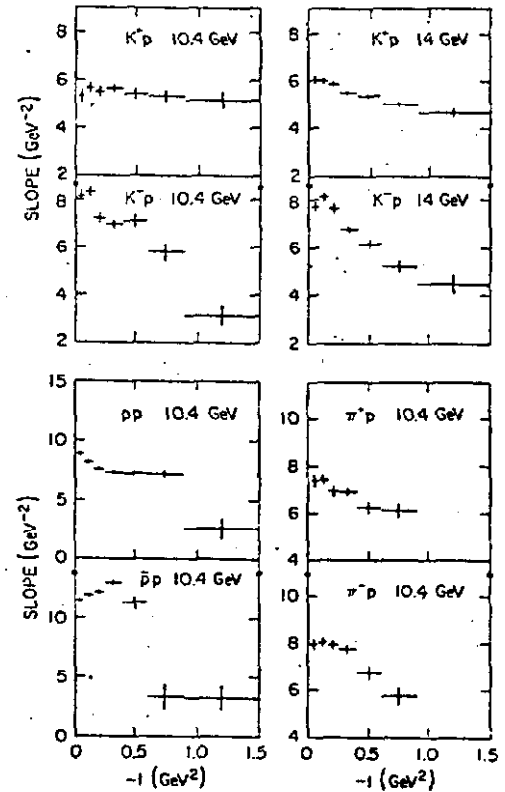


Figure 12: b as a function of t from Ref. 3.

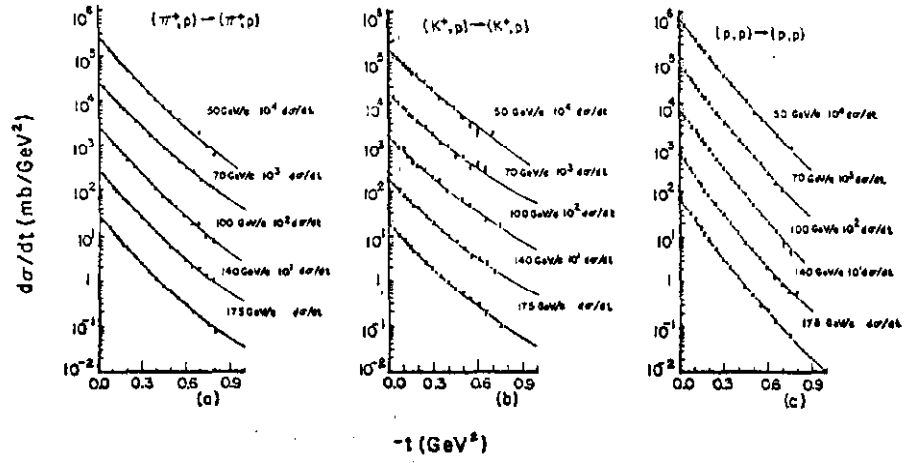


Figure 13: $d\sigma/dt$ for π^+p , K^+p , pp from Ref. 13.

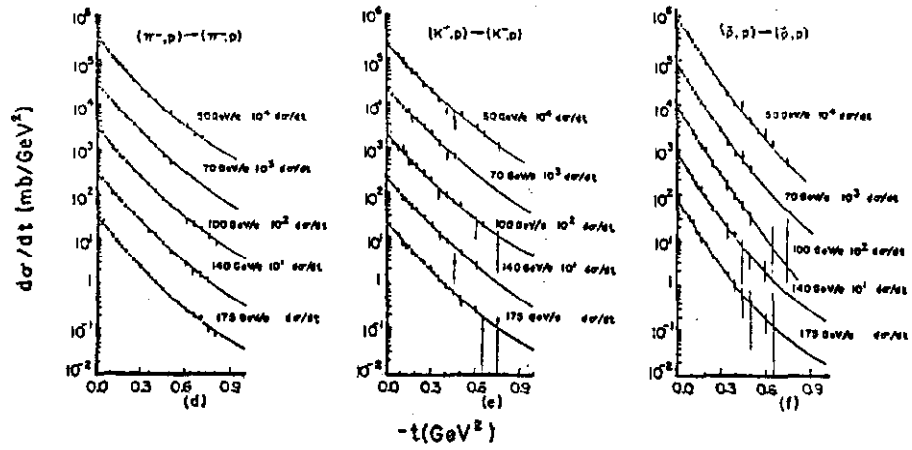


Figure 14: $d\sigma/dt$ for π^-p , K^-p , $\bar{p}p$ from Ref. 13.

change than allowed by the above form. This occurred for $\pi^\pm p$ and pp at 200 GeV/c near $t = -0.40 \text{ GeV}^2$. Other than this the two experiments are in reasonable agreement as they also are in regions of kinematical overlap with Fermilab E-69.

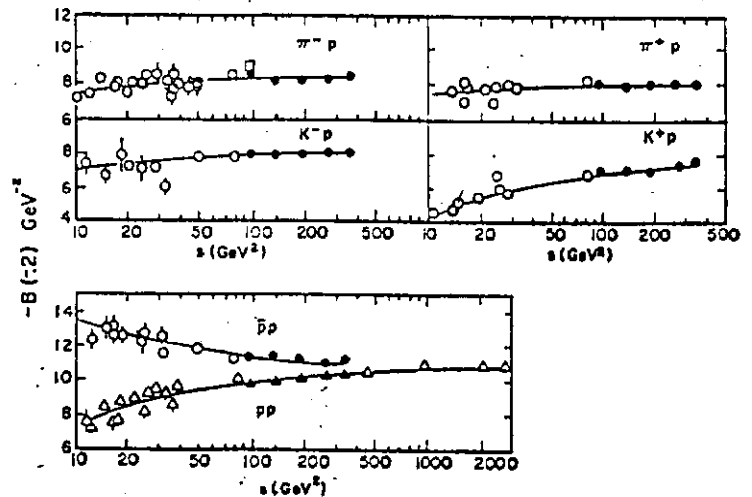


Figure 15: s -dependence of $b(-0.2)$ for π^\pm , $K^\pm p$ and $p^\pm p$ from Ref. 13.

A standard way to look at the s dependence of these forward slopes is to plot the logarithmic derivative, $b(t)$ at a particular value of t as a function of s . This helps to minimize the sensitivity to the t range and the fitting procedure. Figure 15 shows a recent compilation by the Fermilab Single Arm Spectrometer Group.¹³ The extension of the Pomeranchuk theorem to the differential cross sections require that the particle and antiparticle slopes should become equal at large s . Indeed the pions and kaons have come close to having a common slope of about 8 GeV^{-2} . The proton and antiproton are still some distance apart but seem to be approaching a value of about 10.5 GeV^{-2} but from opposite directions.

The data of the Fermilab Single Arm Spectrometer Group provides us with an opportunity to make some simple Regge model comparisons since it includes six incident particles and a substantial s range. These comparisons have been made by the group in reference 13 and a review article by J. Butler.¹⁵ Figure 16 is a table of the parameters to a fit by a single effective Regge pole with trajectory $\alpha(t)$.

$$\frac{d\sigma}{dt} = A_{s_0} e^{(B_{s_0} t + C_{s_0} t^2)} (s/s_0)^{2(\alpha_0 + \alpha' |t|) - 2}$$

	A_{s_0} (mb/GeV ²)	B_{s_0} (GeV ⁻²)	C_{s_0} (GeV ⁻⁴)	α_0	α' (GeV ⁻²)	χ^2/deg
π^+p	28.09 \pm .24	8.89 \pm .07	2.12 \pm .21	1.013 \pm .005	-.057 \pm .021	93/81
K^+p	18.29 \pm .30	8.20 \pm .14	2.09 \pm .24	1.059 \pm .012	-.147 \pm .053	65/69
pp	74.88 \pm .69	10.77 \pm .08	1.88 \pm .14	0.981 \pm .006	-.203 \pm .025	95/76
π^-p	29.58 \pm .23	9.39 \pm .06	2.66 \pm .10	0.990 \pm .005	-.036 \pm .020	157/78
K^-p	21.13 \pm .49	8.94 \pm .23	2.48 \pm .41	1.012 \pm .017	-.059 \pm .079	66/61
$\bar{p}p$	86.18 \pm 1.75	12.34 \pm .22	2.96 \pm .50	0.924 \pm .015	-.167 \pm .091	95/62

Figure 16: Fermilab Single Arm Spectrometer data¹³

fit to a simple Regge pole.

My only point is that this table makes it clear that these reactions cannot be dominated by the same effective pole since the reactions have differing slopes, α' . One would expect that the two reactions which are exotic in the s-channel, K^+p and pp , might be dominated by Pomeron exchange at lower energy than the rest and indeed they both have $\alpha' \approx -0.2 \text{ GeV}^{-2}$.

The above single Regge pole analysis is clearly too simple at these energies and to make it more complete we would add to the Pomeron contribution, contributions from the f , A_2 , ρ , and ω exchanges. The contribution to the elastic amplitude from these poles is given in Fig. 17. Note that the exchanges with $C = -1$, change sign as one goes from particle to antiparticle scattering. The fact that the π^+p and π^-p distributions are almost identical at these energies implies that the ρ exchange term is small.

Another piece of data relevant to the $C = -1$ amplitude is the cross over point, t_c . This is the value of t

where $d\sigma/dt (x^+p) = d/dt (x^-p)$.

Figure 18 shows the s dependence of t_c and the new data of the Fermilab Single Arm Spectrometer Group. This quantity becomes very hard to measure at high energies and the new points are inconclusive although perhaps t_c for pp is decreasing. Optical models prefer it to get smaller whereas simple Regge models would like it to be independent of s .

There is a great deal more that can be said about the very rich data of the Fermilab Single Arm Spectrometer Group such as the extraction of the ϕp cross sections and their impact parameter analysis. These are well described in their publications.¹³

As we go out to larger t we observe a new set of phenomena that is illustrated in Fig. 19. The K^+p and π^-p elastic cross sections are indistinguishable at Fermilab energies for t beyond about

$$\begin{aligned}
 f(pp) &= f_p^{pp} + f_f^{pp} + f_{A_2}^{pp} - f_\rho^{pp} - f_\omega^{pp} \\
 f(\bar{p}p) &= f_p^{pp} + f_f^{pp} + f_{A_2}^{Kp} + f_\rho^{pp} + f_\omega^{pp} \\
 f(K^+p) &= f_p^{Kp} + f_f^{Kp} + f_{A_2}^{Kp} - f_\rho^{Kp} - f_\omega^{Kp} \\
 f(K^-p) &= f_p^{Kp} + f_f^{Kp} + f_{A_2}^{Kp} + f_\rho^{Kp} + f_\omega^{Kp} \\
 f(\pi^+p) &= f_p^{\pi p} + f_f^{\pi p} - f_\rho^{\pi p} \\
 f(\pi^-p) &= \underbrace{f_p^{\pi p} + f_f^{\pi p}}_{C = +1} + \underbrace{f_\rho^{\pi p}}_{C = -1}
 \end{aligned}$$

Figure 17: Contribution to elastic amplitudes.

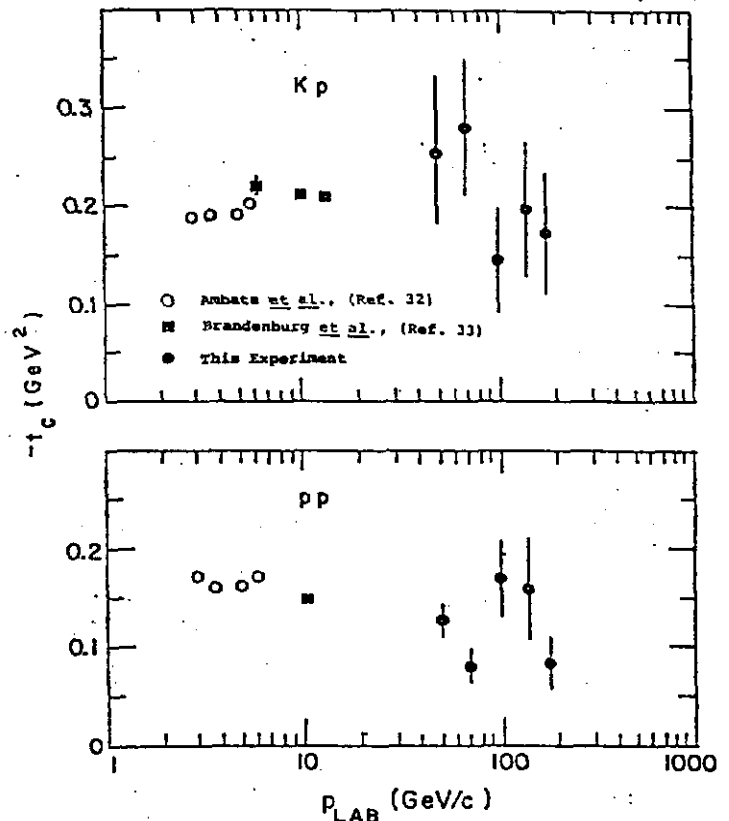


Figure 18: t_c , the cross over point as a function energy for pp and Kp from Ref. 13.

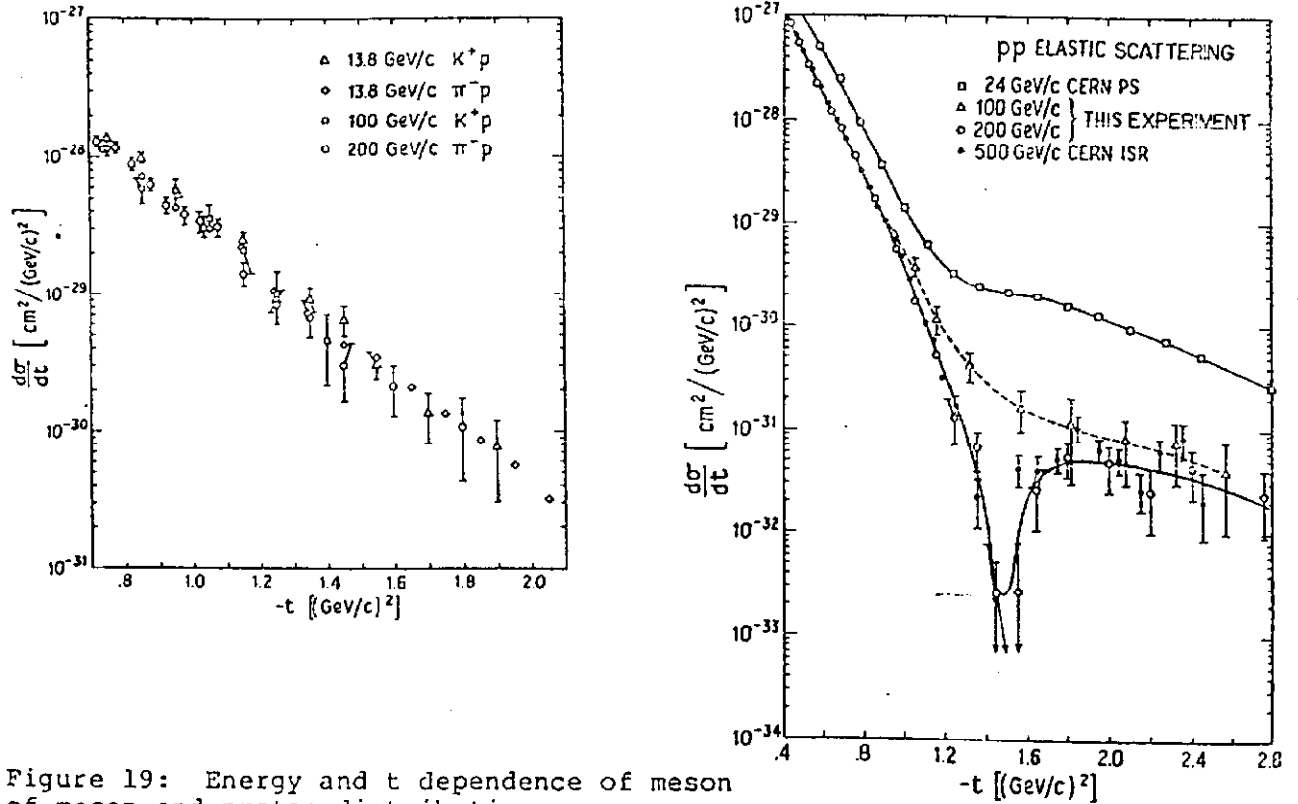


Figure 19: Energy and t dependence of meson of meson and proton distributions near the pp dip at $t = -1.4$ GeV. These figures are from Ref. 14.

-0.8 GeV^2 . The K^-p and π^+p show similar behavior. This data comes from the Fermilab E-7 group,¹⁴ and they conjecture that the core region for Kp and πp interaction behave similarly and the differences manifested by the low t behavior come from the peripheral regions.

The pp distributions¹⁴ of Fig. 19 are very different from the meson case. The dip which was first seen in ISR experiments¹⁶⁻¹⁸ is seen to have a rather abrupt onset in the Fermilab momentum range. Its s dependence over the ISR energy has been studied in some detail¹⁷ and is illustrated in Fig. 20. Little s dependence is seen in the slope for the range $0.25 < |t| < 0.60$ but for values of $|t|$ just below the dip, the slope does get steeper with increasing s . Beyond the dip there again seems to be little s dependence of the slope. The dip does move to lower $|t|$ with increasing energy as expected for diffraction on an object of increasing radius. Data made available by the same Split

Field Magnet group¹⁸ last year carried these measurements out to $t \approx -9 \text{ GeV}^2$ and is shown in Fig. 21. Note that there are no more dips and the logarithmic slope has a constant value⁺ of 1.8 GeV^{-2} beyond $|t| = 2.0 \text{ GeV}^2$.

The s and t dependence of $d\sigma/dt$ in the region of the dip is well described by a form

$$\frac{d\sigma}{dt} = \left| \sqrt{A} \exp(Bt/2) + \sqrt{C} \exp(Dt/2 + i\phi) \right|^2$$

suggested by Phillips and Barger.¹⁹ Figure 20 shows the fits to this two amplitude form at two ISR energies. The C and D parameters were kept constant and s dependence given to the other parameters. Thus the simple model of two interfering amplitudes, one having no energy dependence, gives a reasonable phenomenological description of the dip behavior over the ISR range.

⁺Note added in proof: New data presented at this meeting by the same group shows a flattening of the slope at very large $|t|$.

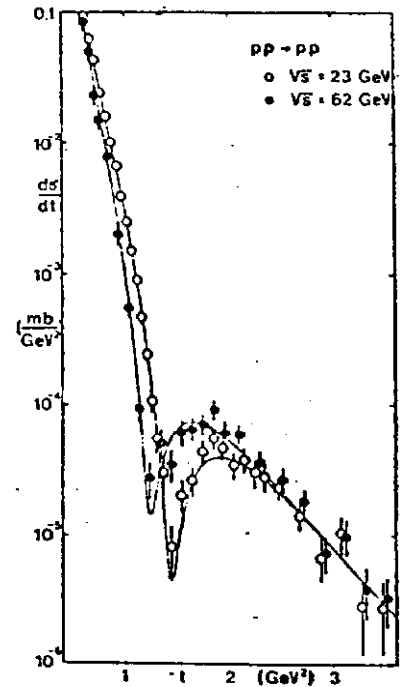


Figure 20: The dip region at two different ISR energies from Ref. 17.

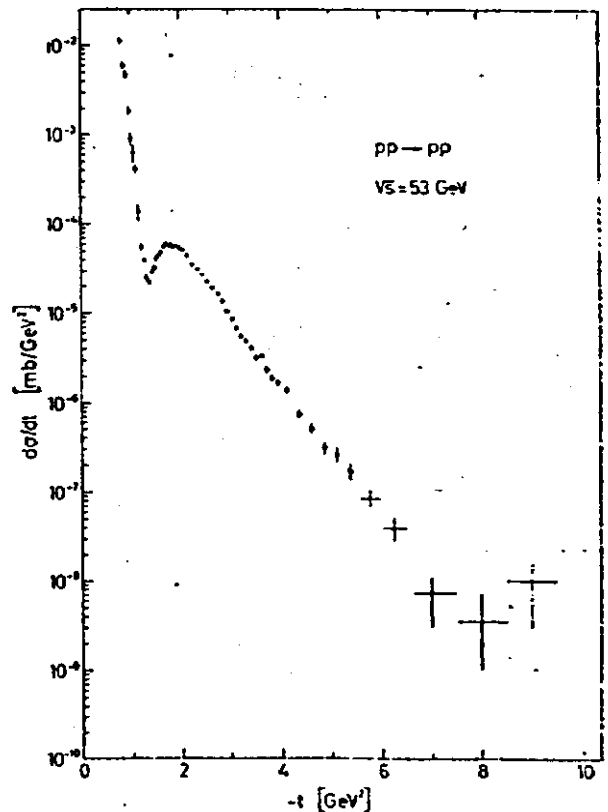


Figure 21: $d\sigma/dt$ behavior at large $|t|$ from Ref. 18.

These recent ISR experiments which go out to large s and t are presumably in a region where the peripheral component has died away. Here we might be able to test the prediction of the constituent interchange model²⁰ which claims

$$\frac{d\sigma}{dt} \rightarrow s^{-10} f(\theta^*)$$

A fit to lower energy data by Landshoff and Polkinghorne²¹ showed reasonably good agreement to this form. Recently however C. Hojvat and J. Orear²² have looked at new data in the light of this model and find an s dependence which is shallower and in poor agreement as shown in Fig. 22. Note however that Landshoff and Polkinghorne were looking at $\theta^* > 30^\circ$ whereas the fit of Hojvat and Orear is at $\theta^* = 4.85^\circ$.⁺ They do find that the data over a wide range of s appear to be linear in a simple p_\perp plot as shown in Fig. 23.

⁺ Note added in proof: S. Brodsky pointed out that this is a crucial difference.

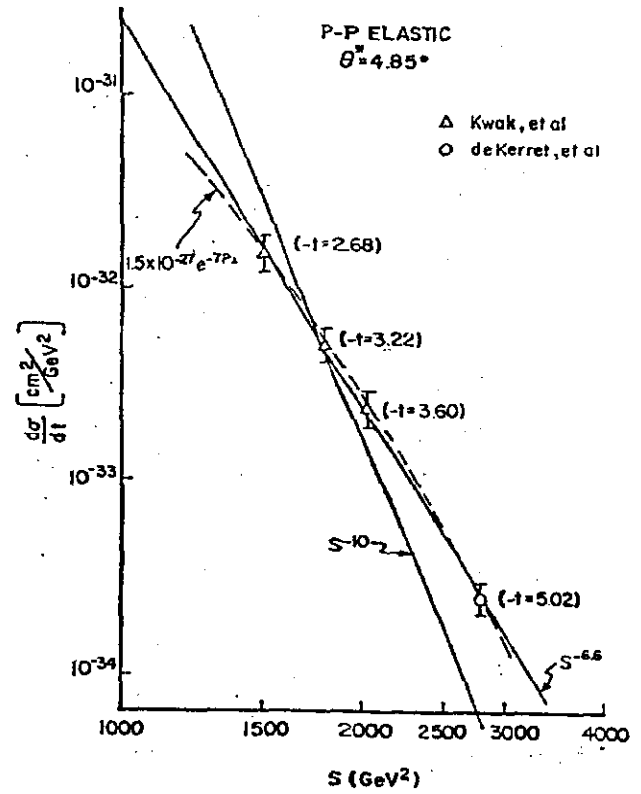


Figure 22: $d\sigma/dt$ versus s at fixed θ^* from Ref. 22.

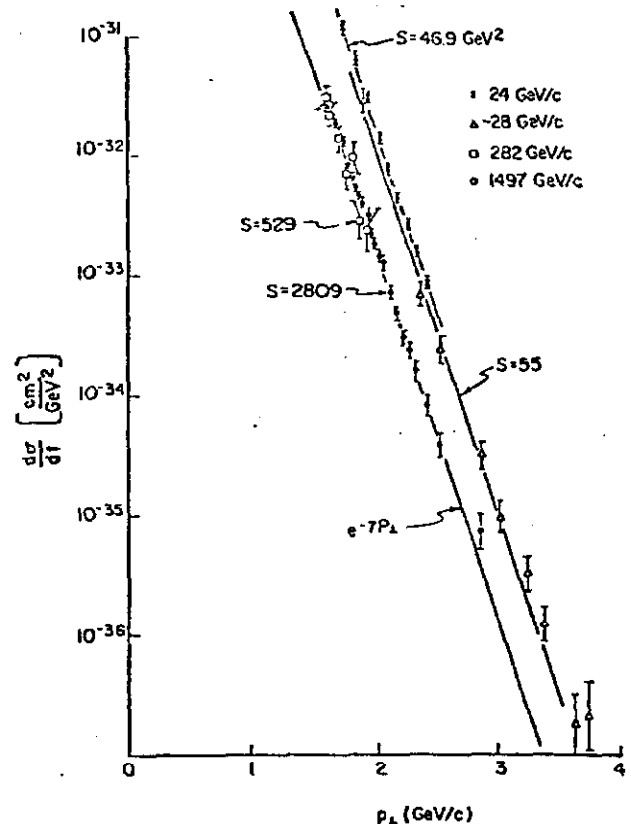


Figure 23: $d\sigma/dt$ versus p_\perp from Ref. 22.

I would like to show some new data from Fermilab on polarization in pp elastic scattering. Although most models assume that pp elastic scattering is governed by the imaginary part of the non-flip amplitude, there are really 5 amplitudes that can contribute. Polarization measurements give us a handle on these other amplitudes however.

The University of Indiana group (E-313) running in the Internal Target area are able to sit at a fixed t and look at the s -dependence of the recoil proton polarization. Figure 24 and 25 show their preliminary results at $|t| = 0.3$ and 0.8 GeV^2 . It is clear that the polarizations are small at large s but for $t = 0.8 \text{ GeV}^2$ it may indeed be turning negative. A complete analysis of in hand data should reduce the errors on many of their points. The Indiana group is able to go out to a little large t but not to the dip region.

Results from Fermilab E-61 (polarized target group) are now starting to come in. This group is now running in the region of the dip at $|t| = 1.4 \text{ GeV}^2$ and at 300 GeV. This same group has data on π^+p at 100 GeV/c and the polarizations they see are small.

Elastic scattering is a big topic and I've tried to cover some of the new developments of the last year or two. My selection of topics has, I'm afraid, reflected my own interests and I hope also some of yours.

I am indebted to D. Leith, members of the Stonybrook and University of Indiana groups for allowing me to use their data before publication. I would also like to acknowledge useful conversations with Joel Butler and my colleagues on E-69.

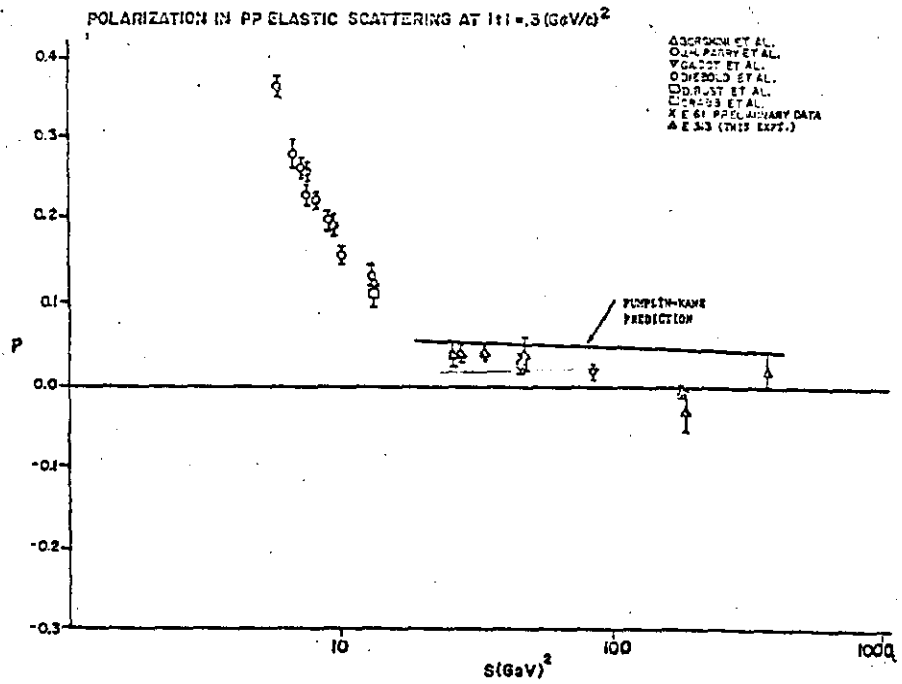


Figure 24: Polarization in pp elastic scattering at $|t| = 0.3 \text{ GeV}^2$ from Ref. 23.

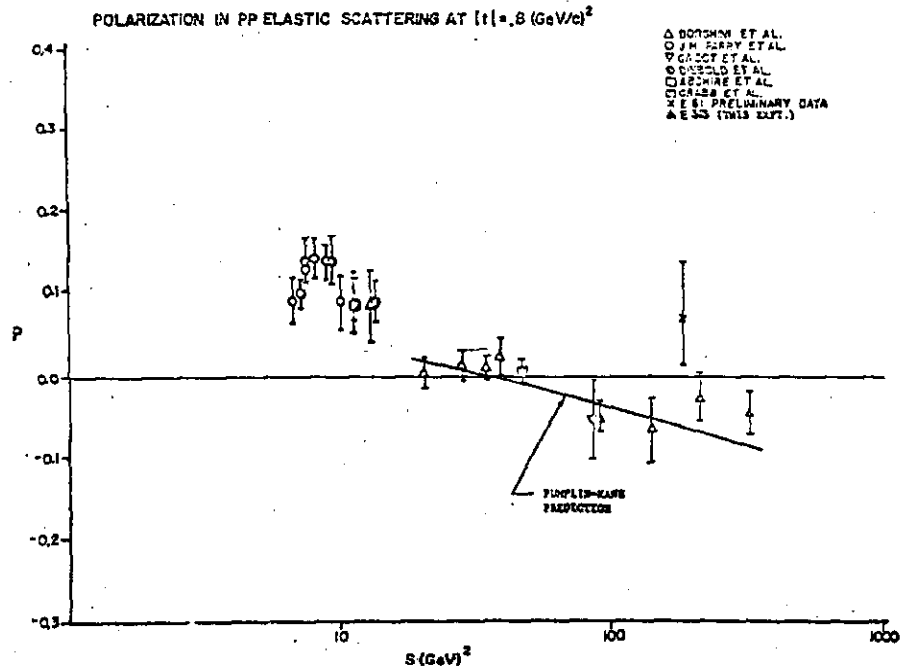


Figure 25: Polarization in pp elastic scattering at $|t| = 0.8 \text{ GeV}^2$ from Ref. 23.

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